One-way ANOVA in a Nutshell

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Definitions

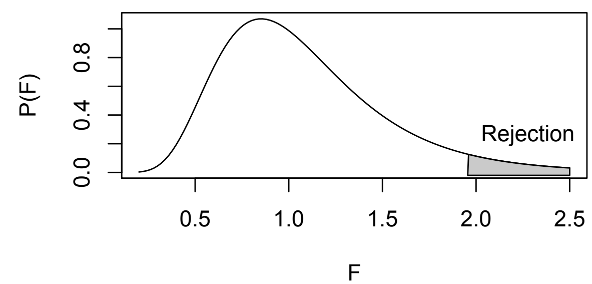
X_{ij}	observations (measurements) for repeated measure i in group j
c	number of groups (categories)
n_j	number of measurements in category j
$n = \sum_{j=1}^{c} n_j$	total measurements in the sample (all categories)
$\overline{X}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} X_{ij}$	mean response for group c
$\overline{\overline{X}} = \frac{1}{n} \sum_{j=1}^{c} \sum_{i=1}^{n_j} X_{ij}$	mean of all responses, a.k.a. the *Grand Mean* of the data.

Problem Definition

1. Define the Null Hypothesis

$$H_0: \qquad \mu_1 = \mu_2 = \cdots = \mu_c$$
$$H_1: \qquad \mu_i \neq \mu_j \quad \text{for some at least one pair } i, j\text{: } i \neq j$$

2. Choose the significance α. Note that the F-test is one sided



3. For and degrees of freedom in the sample, compute the critical F: $F_U = F_{\alpha,c-1,n-c}$

Sums of Squares

$$\mathrm{SSA} = \sum_{j=1}^{c} n_j \left(\overline{X}_j - \overline{\overline{X}} \right)^2 \qquad \mathrm{MSA} = \frac{\mathrm{SSA}}{c-1}$$

$$\mathrm{SSW} = \sum_{j=1}^{c} \sum_{i=1}^{n_j} \left(X_{ij} - \overline{X}_j \right)^2 \qquad \mathrm{MSW} = \frac{\mathrm{SSW}}{n-c}$$

$$\mathrm{SST} = \sum_{j=1}^{c} \sum_{i=1}^{n_j} \left(X_{ij} - \overline{\overline{X}} \right)^2 \qquad \mathrm{MST} = \frac{\mathrm{SST}}{n-1}$$

ANOVA Summary Table

Source	Degrees of Freedom	Sums of Squares	Mean Square (Variance)	F
Among Groups	$c-1$	SSA	$\text{MSA} = \dfrac{\text{SSA}}{c-1}$	$F = \dfrac{\text{MSA}}{\text{MSW}}$
Within Groups	$n-c$	SSW	$\text{MSW} = \dfrac{\text{SSW}}{n-c}$	
Total	$n-1$	SST		

From Table 10.1 in Levine, Ramsey and Smidt