ME 448/548   Problem Set 1   Winter 2012

due 24 January 2012

1. (5 points) Estimate the diffusion time scale for thermal energy transport through an oak table 1.5 cm thick if the thermal diffusion coefficient is $1.8 \times 10^{-7}\,\mathrm{m^2/s}$. Estimate the transit time for a sound pulse through a 1.5 cm layer of oak if the sound speed in oak is 3800 m/s. How does time for transmission of a sound wave across a layer of oak compare to the time it takes that same layer to reach steady state after a step change in thermal boundary condition?

2. (10 points) What are the minimum number of time steps $n_{t,\mathrm{min}}$ needed for stable solutions of the heat equation with $\alpha = 0.1$, $L = 1$, $t_{\mathrm{max}} = 1$ if nx $= 51, 101, 201, 401, 801, 1601$?

3. (10 points) Using Example 14.3 as a model, verify that the truncation error BTCS scheme is $\mathcal{O}(\Delta x^2)$ when $\Delta t \sim \Delta x^2$ as Δx is reduced. Repeat the convergence study using $\Delta t = \Delta x$ while Δx is reduced. Is the truncation error dependent on how both Δt and Δx are reduced? Why or why not?

4. (20 points) An aluminum rod 10 cm long and 0.5 cm in diameter is embedded in a large block of expanded polystyrene foam. The rod and foam are initially at a uniform temperature of 21°C. At $t = 0$, two water jets are directed at the exposed ends of the Aluminum rod. One jet has a temperature of 5°C and the other jet has a temperature of 40°C.

$L = 10$ cm
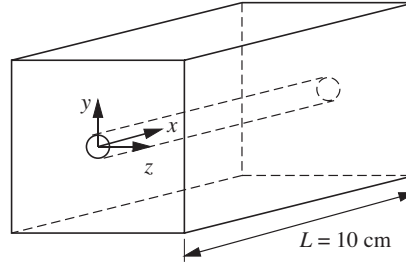
Make a copy of the demoHotPot.m code and call it demoHotRod. Make changes to the demoHotRod code so that the fluid temperatures are $T_{f,0} = 5\,°\mathrm{C}$ at the $x = 0$ end, and $T_{f,L} = 40\,°\mathrm{C}$ at the $x = L$ end, and the initial temperature is $T_0(x) = 21\,°\mathrm{C}$ (a constant). Change the material properties to those of a common aluminum alloy (you choose one). Change the physical dimensions in the code to be consistent with the geometry of the rod. Set the maximum time for the simulation to 90 seconds.

   a. Run the simulation twice: once for a convection coefficient $h = 10\,\mathrm{W/m^2°C}$ on both ends, and again for $h = 10^4\mathrm{W/m^2°C}$ on both ends. Include the final plots for these two runs in your solution.

   b. What is the physical effect of changing h from 10 to 10^4 and how is that effect evident in the results.

   c. Discuss the value of the temperature at the ends of the rod during the two solutions. Does the behavior from the simulation make sense? How does this behavior differ from imposing constant temperatures of $T_{f,0} = 5\,°\mathrm{C}$ at the $x = 0$ end, and $T_{f,L} = 40\,°\mathrm{C}$ at the $x = L$ end?

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5. (20 points) Consider the analytical solution to

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le L, \quad t > 0 \tag{1}$$

$$u(x,0) = c_1 - c_2 x \tag{2}$$

$$u(0,t) = u(L,t) = 0 \tag{3}$$

where α, c_1 and c_2 are constants.

   a. What is the formula for the constants, A_n, in the analytical solution?

   b. What are the first five terms, A_n, $n = 1, 2, \ldots, 5$ when $\alpha = 0.1$, $L = 5$, $c_1 = 2$, and $c_2 = 0.4$.