Temperature Measurement

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Temperature Measurement

- Liquid bulb thermometers
- Gas bulb thermometers
- bimetal indicators
- RTD: resistance temperature detectors (Platinum wire)
- thermocouples
- thermistors
- IC sensors
- Optical sensors
  - ▷ Pyrometers
  - ▷ Infrared detectors/cameras
  - ▷ liquid crystals

IC Temperature Sensors

- Semiconductor-based temperature sensors or thermocouple reference-junction compensation
- Packaged suitable for inclusion in a circuit board
- Variety of outputs: analog (voltage or current) and digital
- More useful for a manufactured product or as part of a control system than as laboratory instrumentation.

Examples (circa 2006)

Manufacturer	Part number
Analog Devices	AD590, AD22103
Dallas Semiconductor	DS1621
Maxim	Max675, REF-01, LM45
National Instruments	LM35, LM335, LM75, LM78

Thermistors (1)

A thermistor is an electrical resistor used to measure temperature. A thermistor designed such that its resistance varies with temperature in a repeatable way.

A simple model for the relationship between temperature and resistance is

$$\Delta T = k \Delta R$$

A thermistor with $k > 0$ is said to have a *positive temperature coefficient* (PTC). A thermistor with $k < 0$ is said to have a *negative temperature coefficient* (NTC).

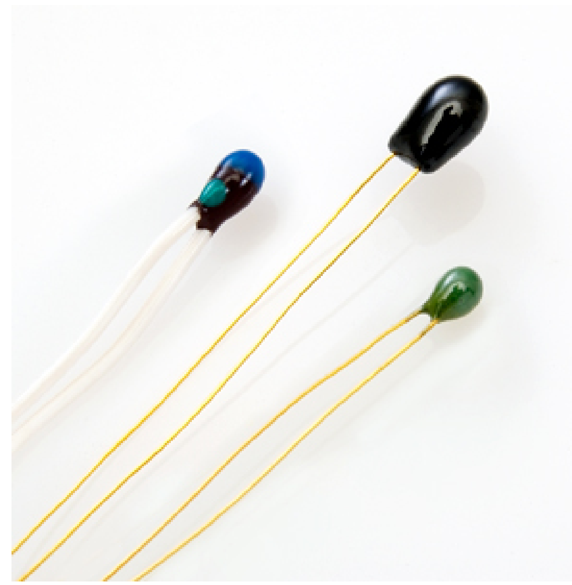


Photo from YSI web site:
`www.ysitemperature.com`

Thermistors (2)

- NTC thermistors are semiconductor materials with a well-defined variation electrical resistance with temperature
- Mass-produced thermistors are interchangeable: to within a tolerance the thermistors obey the same $T = F(R)$ relationship.
- Measure resistance, e.g., with a multimeter
- Convert resistance to temperature with calibration equation

Thermistors (3)

Advantages

- Sensor output is directly related to absolute temperature – no reference junction needed.
- Relatively easy to measure resistance
- Sensors are interchangeable ($\pm 0.5\,^{\circ}\mathrm{C}$)

Disadvantages

- Possible self-heating error
  - ▷ Each measurement applies current to resistor from precision current source
  - ▷ Measure voltage drop, then compute resistance from known current and measured voltage
  - ▷ Repeated measurements in rapid succession can cause thermistor to heat up
- More expensive than thermocouples: \$20/each versus \$1/each per junction
- More difficult to apply for rapid transients: slow response and self-heating

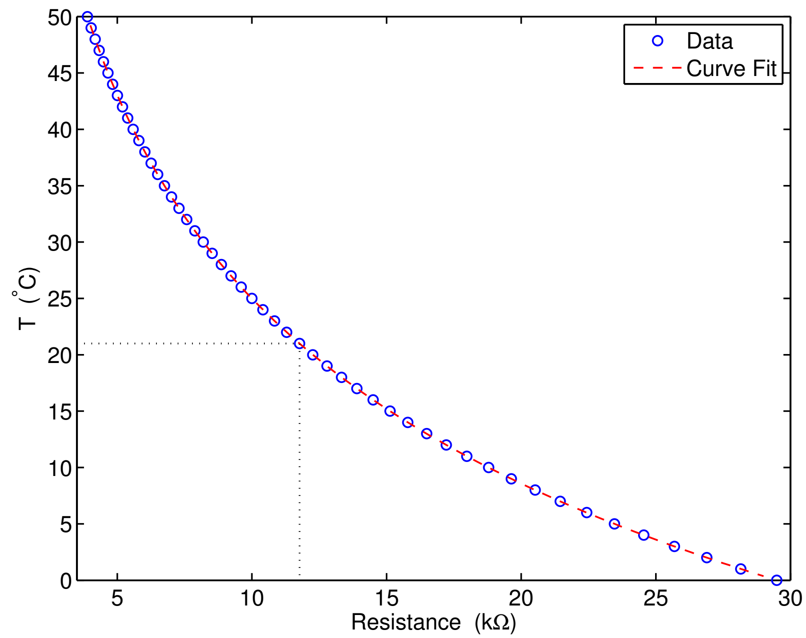
Thermistors (4)

Calibration uses the Steinhart-Hart equation

$$T = \frac{1}{c_1 + c_2 \ln R + c_3(\ln R)^3}$$

Nominal resistance is controllable by manufacturing.

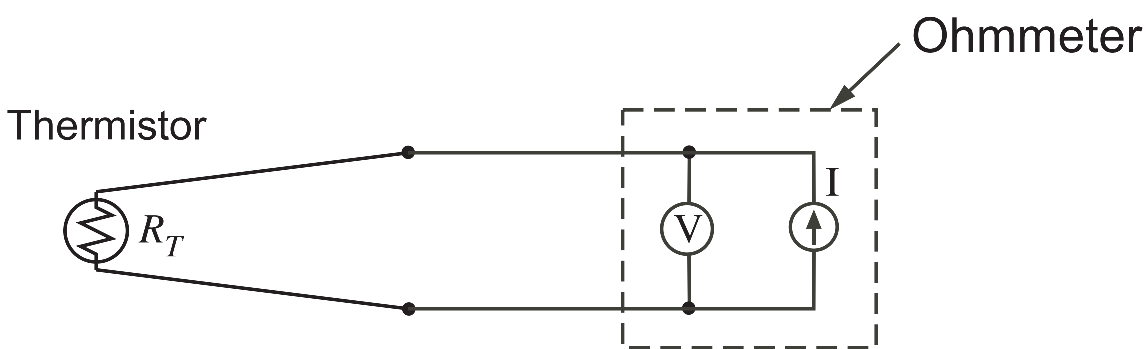
Typical resistances at 21 °C:
10 kΩ, 20 kΩ, ... 100 kΩ.



Thermistors (5)

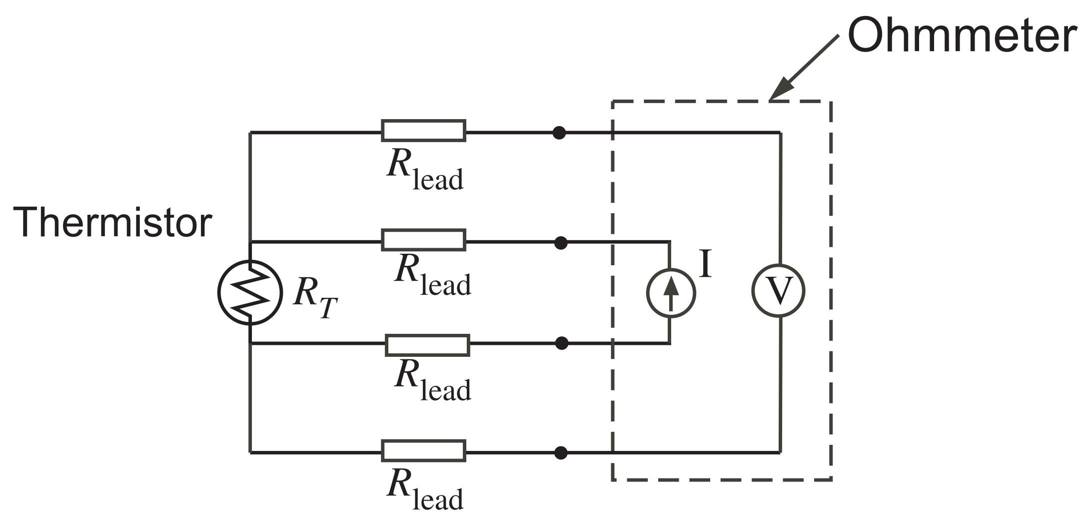
Two-wire resistance measurement: $R_T = \dfrac{V}{I}$.

Resistance in the lead wires can lead to inaccurate temperature measurement.



Thermistors (6)

Four-wire resistance measurement eliminates the lead resistance[1]



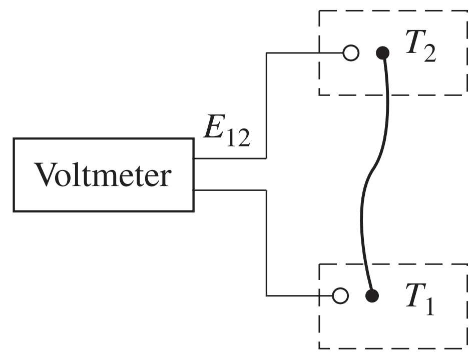
[1]Sketch adapted from *Hints for Making Better Digital Multimeter Measurements*, Agilent Technologies Corporation, `www.agilent.com`.

Thermocouples: Overview

- Principle of operation
- Wire types: B, E, J, K, N, R, S, T
- Formats: prefab, homemade, fast response, slow response
- Circuit diagrams: reference junction compensation
- Good practice

Seebeck Effect (1)

Temperature gradient in a conductor induces a voltage potential



$$E_{12} = \bar{\sigma}(T_2 - T_1) \tag{1}$$

where $\bar{\sigma}$ is the average *Seebeck coefficient* for the range $T_1 \leq T \leq T_2$.

Seebeck Effect (2)

Perturb T_2 while holding T_1 fixed:

$$E_{12} + \Delta E_{12} = \bar{\sigma}(T_2 - T_1) + \sigma(T_2)\Delta T_2 \tag{2}$$

Subtract Equation (1) from Equation (2) to get

$$\Delta E_{12} = \sigma(T_2)\Delta T_2 \tag{3}$$

Rearrange

$$\sigma(T_2) = \frac{\Delta E_{12}}{\Delta T_2} \tag{4}$$

σ is an intrinsic property of the material, so

$$\sigma(T) = \lim_{\Delta T \to 0} \left(\frac{\Delta E}{\Delta T} \right) \tag{5}$$

Seebeck Effect (3)

Applying the limit yields the derivative:

$$\lim_{\Delta T \to 0} \left(\frac{\Delta E}{\Delta T} \right) = \frac{dE}{dT}$$

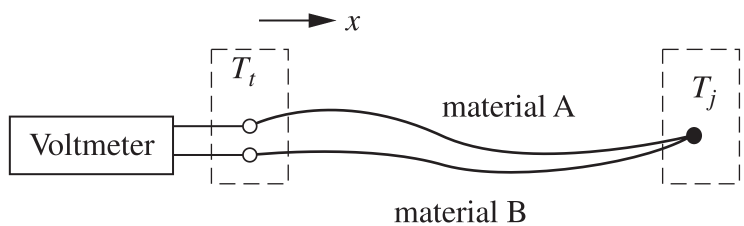
so

$$\boxed{\sigma(T) = \frac{dE}{dT}} \tag{6}$$

Equation (6) is the *definition of the Seebeck Coefficient*

EMF Relationships for Thermocouples (1)

A simple thermocouple:



Rewrite Equation (6) as

$$dE = \sigma(T)\, dT \tag{7}$$

Thus, the emf generated in material A between the junction at T_t and the junction at T_j is

$$E_{A,tj} = \int_{T_t}^{T_j} \sigma_A(T) dT \tag{8}$$

Applying Equation (8) to consecutive segments of the circuit gives

$$E_{AB} = \int_{T_t}^{T_j} \sigma_A\, dT + \int_{T_j}^{T_t} \sigma_B\, dT \tag{9}$$

EMF Relationships for Thermocouples (2)

Switch limits of integration

$$E_{AB} = \int_{T_t}^{T_j} \sigma_A \, dT - \int_{T_t}^{T_j} \sigma_B \, dT = \int_{T_t}^{T_j} (\sigma_A - \sigma_B) \, dT \tag{10}$$

Define the Seebeck coefficient for the *material pair* AB as

$$\sigma_{\mathrm{AB}} = \sigma_A - \sigma_B \tag{11}$$

Then

$$E_{AB} = \int_{T_t}^{T_j} \sigma_{\mathrm{AB}} \, dT \tag{12}$$

The emf generated by the Seebeck effect is due to the temperature gradient along the wire. The emf is not generated at the junction between two dissimilar wires.

EMF Relationships for Thermocouples (3)

Nominal values of Seebeck Coefficient

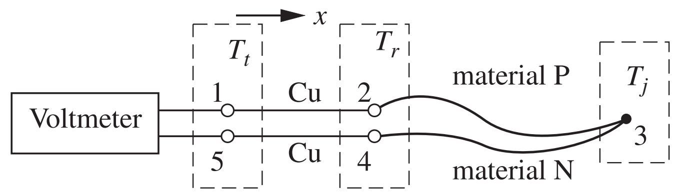
Type	Metal +	Metal −	Seebeck Coefficient	Temperature Range
J	Iron	Constantan	$50\,\mu\mathrm{V}/^\circ\mathrm{C}$	-210 to $+760\,^\circ\mathrm{C}$
K	Nickel-Chromium	Nickel	$39\,\mu\mathrm{V}/^\circ\mathrm{C}$	-270 to $+1372\,^\circ\mathrm{C}$
T	Copper	Constantan	$38\,\mu\mathrm{V}/^\circ\mathrm{C}$	-270 to $+400\,^\circ\mathrm{C}$

σ values are small, so the voltage output from thermocouples is small, typically on the order of 10^{-3} V.

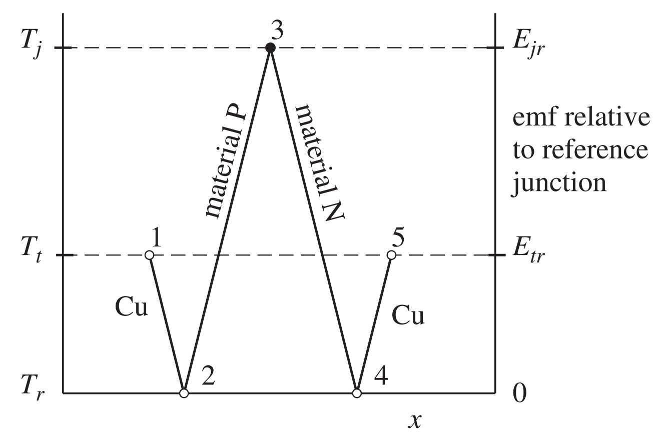
Reference Junction (1)

Thermocouples are only capable of measuring temperature differences. To measure the temperature of an object, we need a known reference temperature. The thermocouple is used to measure the temperature difference between the object and the known reference temperature.

Physical Circuit:



Conceptual $T(x)$ Plot:



Reference Junction (2)

Applying Equation (12) gives

$$E_{15} = \int_{T_t}^{T_r} \sigma_C dT + \int_{T_r}^{T_j} \sigma_P dT + \int_{T_j}^{T_r} \sigma_N dT + \int_{T_r}^{T_t} \sigma_C dT \tag{13}$$

where σ_C is the absolute Seebeck coefficient of copper, σ_P is the absolute Seebeck coefficient of the positive leg, and σ_N is the absolute Seebeck coefficient of negative leg.

Reversing the limits of integration gives

$$\int_{T_t}^{T_r} \sigma_C dT = -\int_{T_r}^{T_t} \sigma_C dT \tag{14}$$

Therefore, the first and last terms in Equation (13) cancel.

$$\Longrightarrow \quad E_{15} = \int_{T_r}^{T_j} \sigma_P dT + \int_{T_j}^{T_r} \sigma_N dT \tag{15}$$

Reference Junction (3)

Reversing the limits on the second integral in Equation (15) gives

$$E_{15} = \int_{T_r}^{T_j} \sigma_P dT - \int_{T_r}^{T_j} \sigma_N dT \qquad = \int_{T_r}^{T_j} (\sigma_P - \sigma_N) dT$$

or

$$E_{15} = \int_{T_r}^{T_j} \sigma_{PN}\; dT \tag{16}$$

where $\sigma_{PN} \equiv \sigma_P - \sigma_N$.

Calibration Curves (1)

Working with calibration data

- Integrals are never evaluated.
- Data is tabulated and curve fit
- Standard polynomial curve fits and coefficients are available for common thermocouple types

Calibration data for T-Type thermocouples:

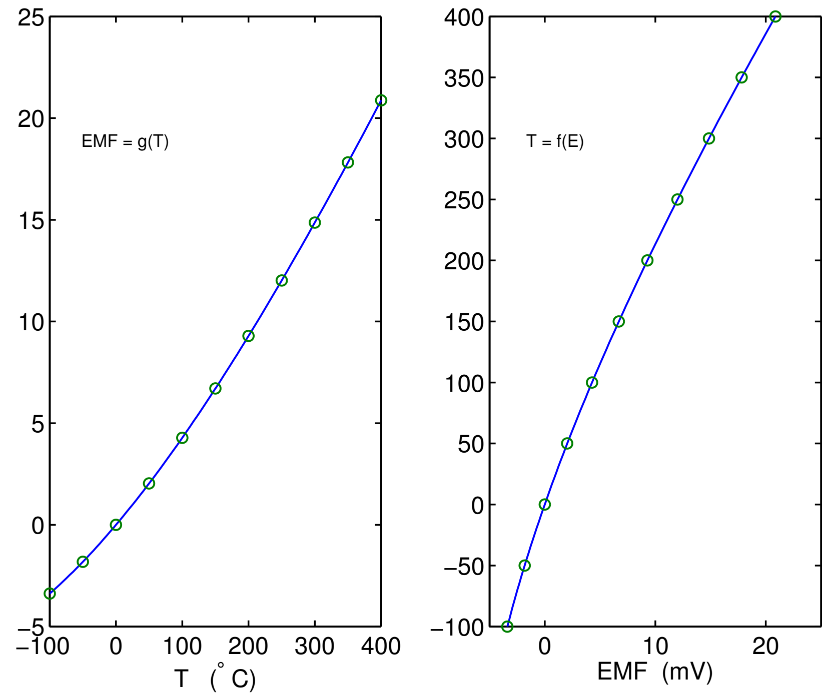
E (mV)	T (°C)
0.0000	0
2.0357	50
4.2785	100
6.7041	150
9.2881	200
12.0134	250
14.8619	300
17.8187	350
20.8720	400

Calibration Curves (2)

ASTM provides standard curve fits to calibration data

$$E_{0j} = b_0 + b_1 T_j + b_2 T_j^2 \ldots b_n T_j^n \quad (17)$$

$$T_j = c_0 + c_1 E_{0j} + c_2 E_{0j}^2 \ldots c_m E_{0j}^m \quad (18)$$



Ice Point Reference Junction

Construction of ice-point reference junction, as recommended by ASTM[2]

134 MANUAL ON THE USE OF THERMOCOUPLES IN TEMPERATURE MEASUREMENT

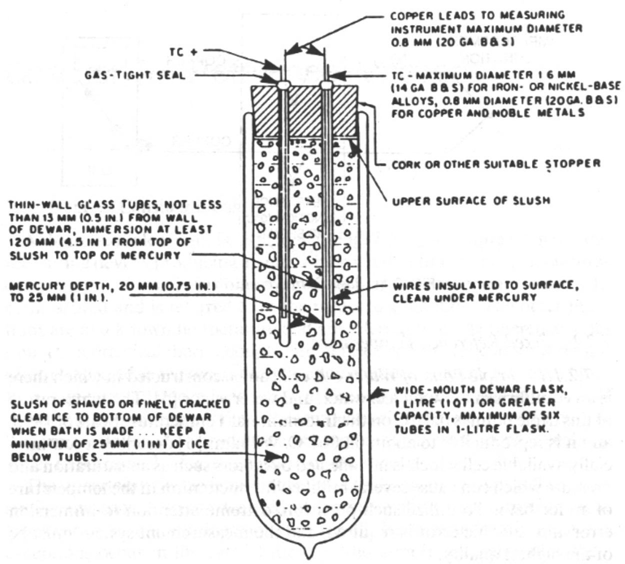


FIG. 7.2—*Recommended ice bath for reference junction.*

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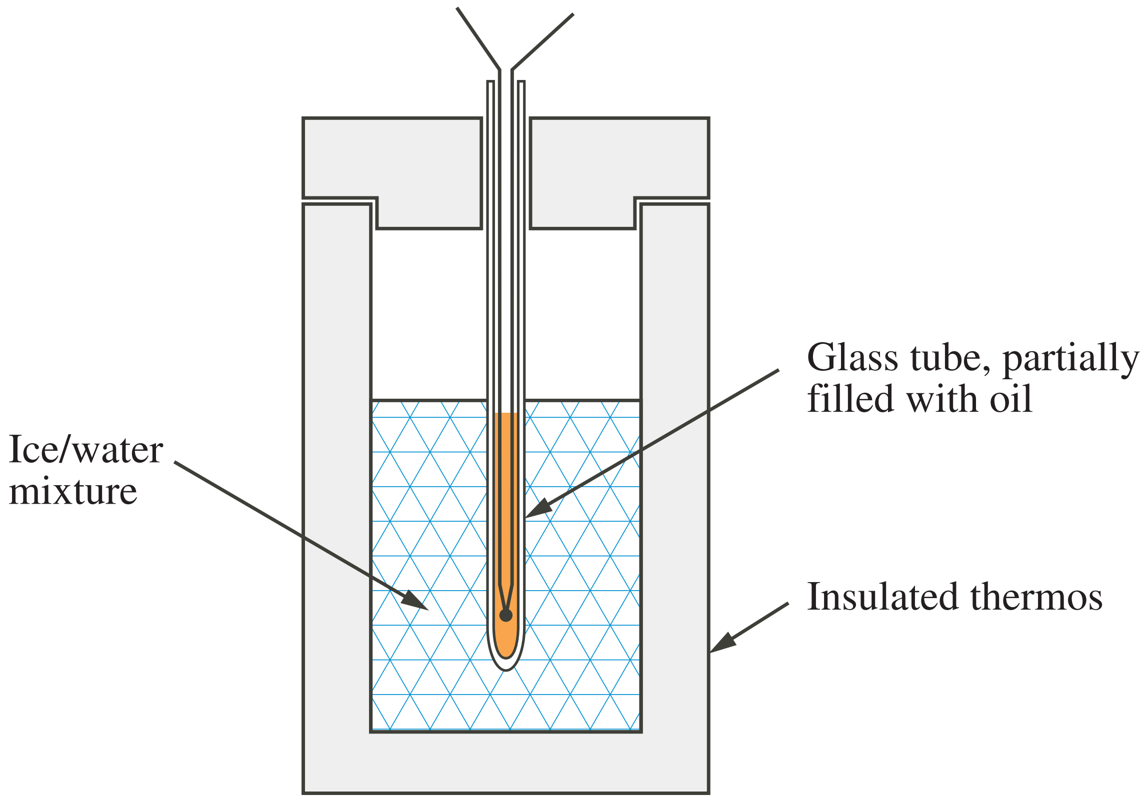
[2]Figure 7.2, p. 134, *Manual on the Use of Thermocouples in Temperature Measurement*, fourth ed., 1993, ASTM, Philadelphia, PA.

Ice Point Reference Junction

Ice-point reference junction used in the thermal lab:



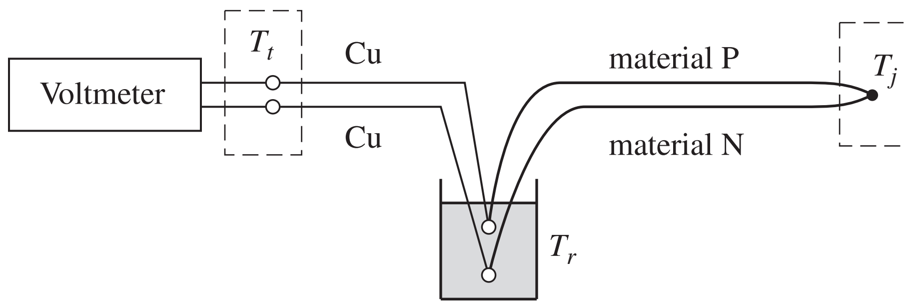
Practical Thermocouple Circuits

Thermocouple circuits used in our laboratory.

- Compensation with a reference junction in an ice bath;
- Compensation with a reference junction at an arbitrary temperature;
- Use of zone boxes for large numbers of thermocouples.

Single Ice-Point Compensation Circuits (1)

Common textbook circuit for ice-point compensation



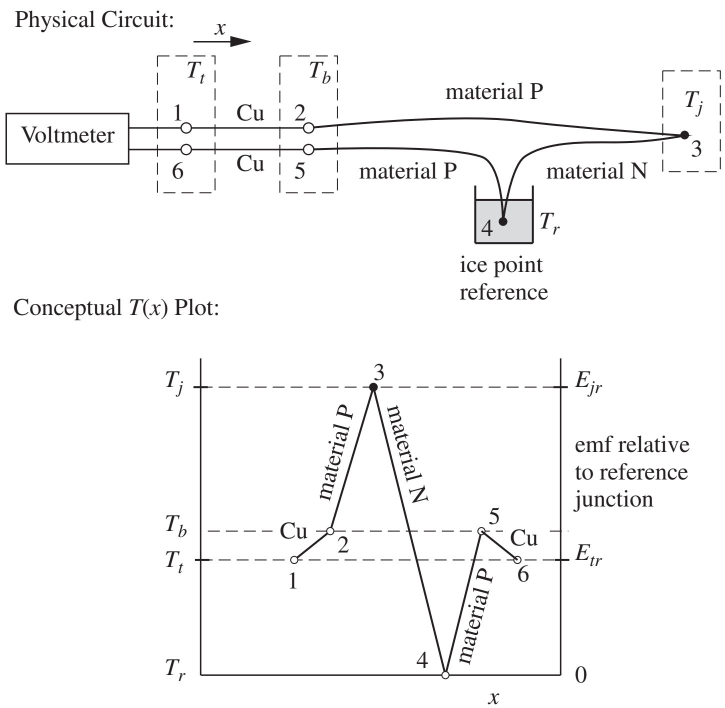
This circuit was analzed in preceding slides. The emf measured by the voltmeter is

$$E = \int_{T_r}^{T_j} \sigma_{PN} \, dT$$

Since T_r is at the melting temperature of ice, the standard calibration equations apply.

Single Ice-Point Compensation Circuits (2)

Alternative ice-point compensation circuit



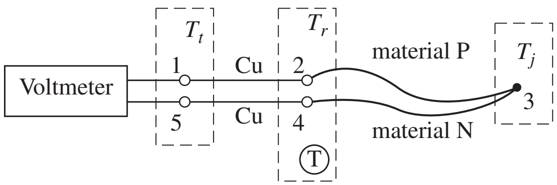
The emf measured by the voltmeter is

$$E = \int_{T_r}^{T_j} \sigma_{PN} \; dT$$

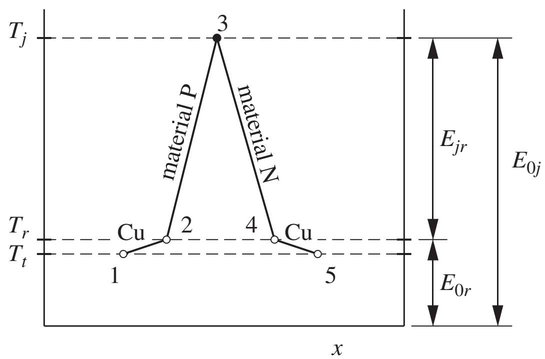
Thus, the standard calibration equations apply to this circuit also.

Thermocouple Conversion for Arbitrary Reference Temperature (1)

Physical Circuit:



Conceptual $T(x)$ Plot:



The emf measured by the voltmeter is

$$E_{rj} = E_{0j} - E_{0r} \qquad (19)$$

where E_{0j} is the emf of a thermocouple with its reference junction at $0\,^{\circ}\mathrm{C}$ and E_{rj} is the output of the thermocouple circuit in the schematic.

Now, since

$$E_{0j} = E_{rj} + E_{0r}$$

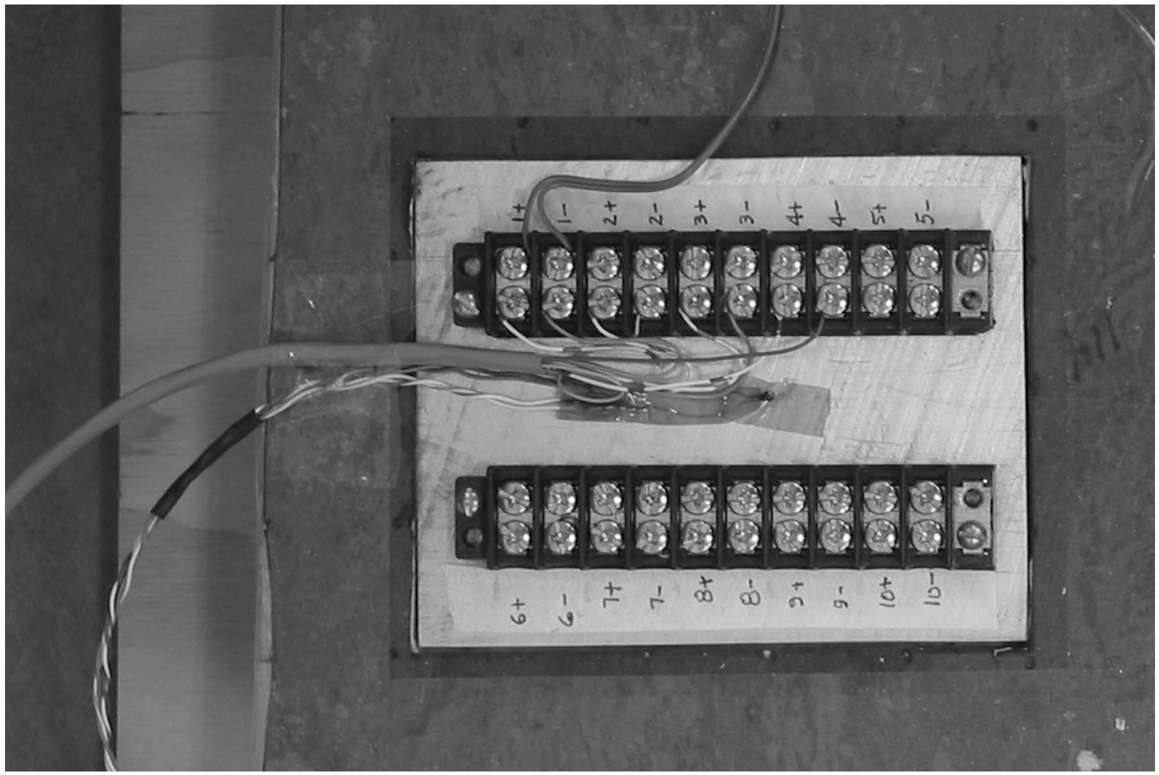
the standard calibration equations apply to this circuit also.

Thermocouple Conversion for Arbitrary Reference Temperature (2)

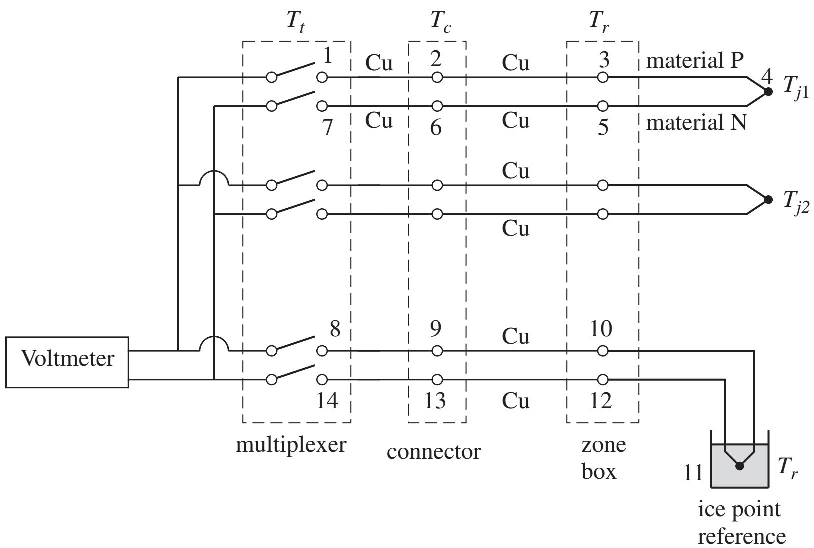
The following steps are used to compute the temperature of a thermocouple with a reference junction at an arbitrary temperature T_r:

1. Measure E_{rj}, the emf of a thermocouple with its reference junction at T_r and its measuring junction at T_j.
2. Compute $E_{0r} = F(T_r)$, the emf of an ice-point compensated thermocouple at temperature T_r.
3. Compute $E_{0j} = E_{rj} + E_{0r}$, the emf of an ice-point compensated thermocouple at T_j.
4. Compute $T_j = F(E_{0j})$ from the ice-point calibration data for the thermocouple.

Zone Box for Reference Junction Compensation (1)



Zone Box for Reference Junction Compensation (2)



The emf output of thermocouple j is

$$E_{0j,1} = \int_0^{T_{j1}} \sigma_{PN} dT$$

Writing the preceding integral as the sum of two integrals and performing simple rearrangments gives

$$E_{0j,1} = -E_{r0} + E_{rj,1} \qquad (20)$$

Therefore, by combining the two electrical measurements of E_{r0} and $E_{rj,1}$, the effective emf of a thermocouple with a reference junction at $0\,^\circ\mathrm{C}$ is obtained.

Zone Box for Reference Junction Compensation (3)

The following steps are used to compute the temperatures of the measuring junctions:

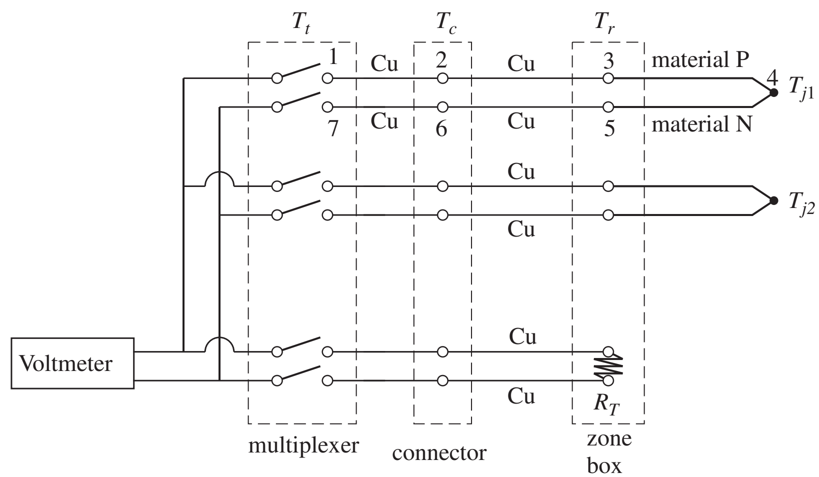
1. Measure E_{r0}, $E_{rj,1}$, $E_{rj,2}$, . . .
2. For each measuring junction, compute

$$E_{0j,i} = E_{rj,i} - E_{r0} \qquad i = 1, \ldots, n$$

   where n is the total number of measuring junctions.
3. Use the thermocouple tables, or Equation (18) to compute the temperature of each junction T_{ji} from $E_{0j,i}$.

Zone Box for Reference Junction Compensation (4)

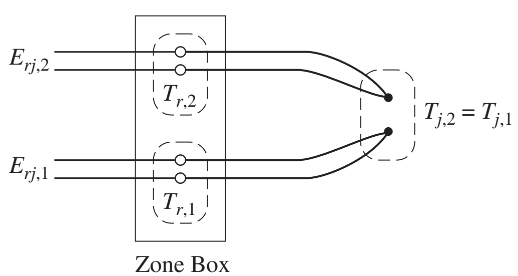
The reference tempeature can be obtained by other means – T_r does not need to be the ice-melting-point. It can be any other temperature, as long as that temperature is known.

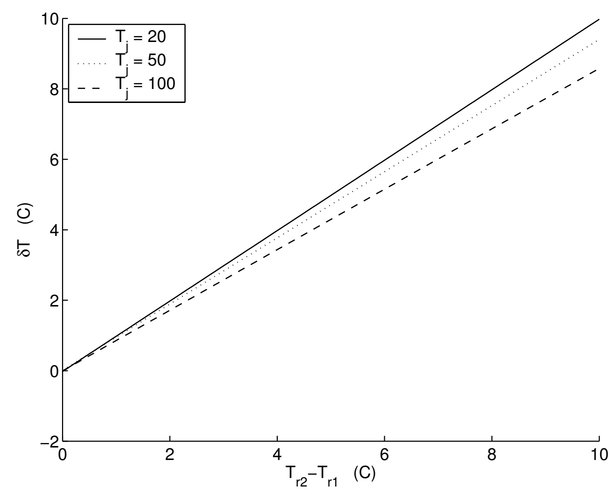


Effect of Non-uniform Zone Box Temperature

A zone box is designed to hold the temperature of all reference junctions at a uniform and easily measured value.

The measurement error introduced by non-uniformities in zone box temperature is roughly linear in he zone box temperature difference: A $0.5^\circ\mathrm{C}$ non-uniformity from junction to junction *in the zone box* causes a $0.5\,^\circ\mathrm{C}$ error in the reading of the measurement junctions.





Zone Box Heat Load from Wiring