# Heat Transfer from a Single, Heated Block

Gerald Recktenwald
Portland State University
Department of Mechanical Engineering
gerry@me.pdx.edu

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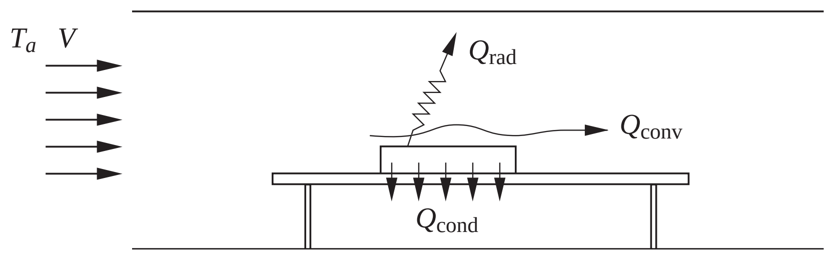
# Overview

- Schematic of Basic Experiment
- Measuring the Heat Transfer Coefficient
- Heater Circuit
- Experiment Planning
- Uncertainty analysis
- Relationship to Practical Thermal Management Problems

# Measurement of Heat Transfer Coefficient (1)




Calculate the heat transfer coefficient from measured quantities

$$h = \frac{Q_{\mathrm{conv}}}{A_s(\bar{T}_s - T_a)} \tag{1}$$

where

| | |
|---|---|
| $T_s =$ | average surface temperature of the block |
| $T_a =$ | temperature of air approaching the block |
| $A_s =$ | surface area of block that is participating in the convection |
| $Q_{\mathrm{conv}} =$ | convective heat loss from the block |

# Measurement of Heat Transfer Coefficient (2)

Not all of the electrical power input is lost directly by convection.

An energy balance on the block gives

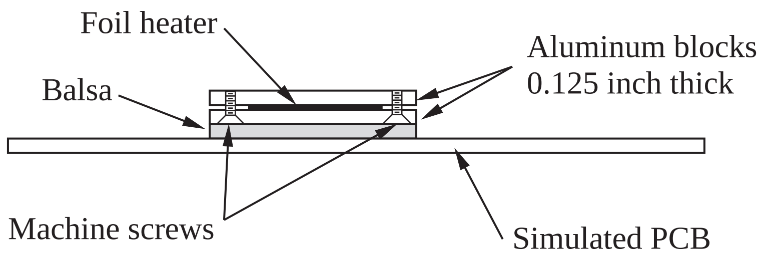
$$Q_{\mathrm{conv}} = \dot{E}_{\mathrm{in}} - Q_{\mathrm{cond}} - Q_{\mathrm{rad}} \tag{2}$$

$Q_{\mathrm{conv}} =$ convective heat loss
$\dot{E}_{\mathrm{in}} =$ electrical power input
$Q_{\mathrm{cond}} =$ conduction heat transfer from block to board
$Q_{\mathrm{rad}} =$ radiation heat transfer to surroundings

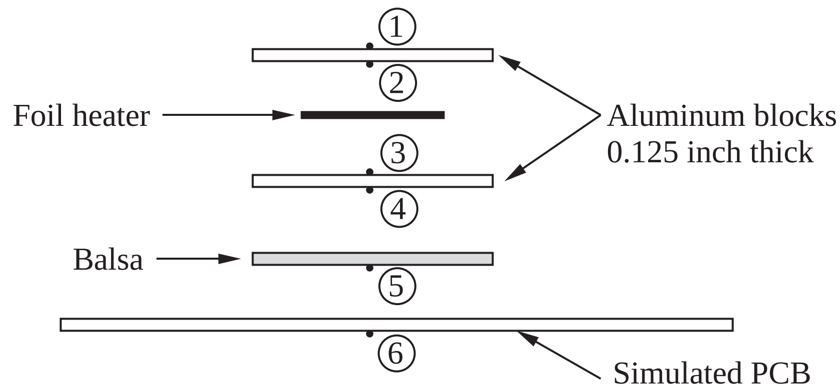
Equation (2) is a centering correction.

# Device Mock-up

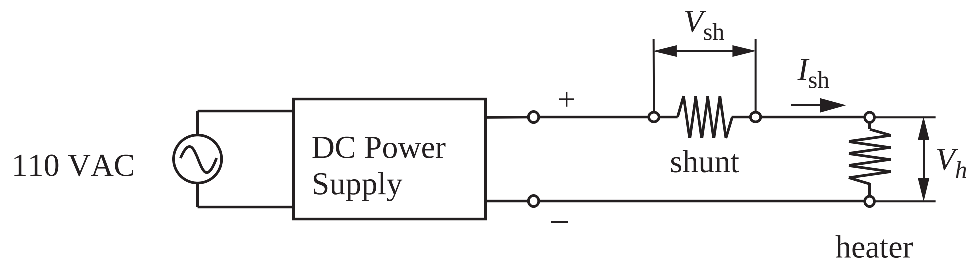
Heater assembly:




Exploded view with potential temperature measurement locations:



# Measuring Power Input (1)




The power dissipated by the electric resistance heater is

$$\dot{E}_{\text{in}} = V_h I_h \tag{3}$$

where $V_h$ is the voltage across the heater, and $I_h$ is the current through the heater. The shunt resistor is used to measure $I_h$. From a measurement of $V_{\text{sh}}$ we compute

$$I_h = I_{\text{sh}} = \frac{V_{\text{sh}}}{R_{\text{sh}}} \qquad \Longrightarrow \qquad \dot{E}_{\text{in}} = \frac{V_h V_{\text{sh}}}{R_{\text{sh}}} \tag{4}$$

## Measuring Power Input (2)

Measuring power with a shunt resistor is much more accurate than measuring just the voltage across the heater and computing $\dot{E}_{\rm in} = V_h^2/R_h$ because $R_h$ varies with temperature.

The shunt resistor has a very low and stable resistance ($R_{\rm sh} \sim 0.005\,\Omega$). Since $R_{\rm sh}$ is very small, the shunt resistor dissipates very little power. Thus its temperature will be stable and its resistance will not change during the experiment.

# Measuring Power Input (3)

**Sample Power Calculation:**

Assume that $R_{\rm sh} = 0.005\,\Omega$ and that $V_{\rm sh} = 2.8 \times 10^{-3}$ V and $V_h = 8.5$ V were measured. Then

$$\dot{E}_{\rm in} = \frac{(8.5)(2.8 \times 10^{-3})}{0.005} = 4.76\,\mathrm{W}$$

and

$$E_{\rm shunt} = \frac{V_{\rm shunt}^2}{R_{\rm sh}}$$

$$= \frac{(2.8 \times 10^{-3})^2}{0.005} = 1.57 \times 10^{-3}\,\mathrm{W}$$

## Experiment Planning

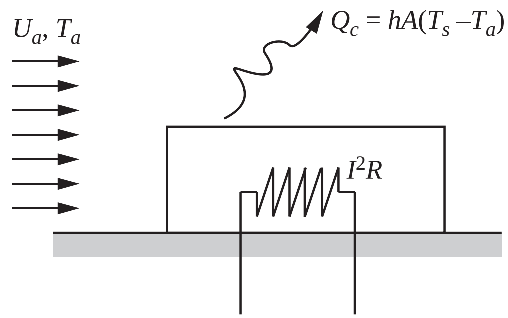
1. Decide which type of sensors, and how many of each type are needed to measure all quantities used to compute $h$ from

$$h = \frac{Q_{\text{conv}}}{A(\bar{T}_s - T_a)}$$

2. Develop a wiring diagram for your experiment.
3. Fabricate thermocouples and attach them to your apparatus.
4. Mount your apparatus to a bottom hatch for the wind tunnel.
5. Route the sensor leads to a terminal strip. Label all wires and document the labeling scheme in your notebook and your report.

# Model of Start-up Transient (1)

How long will you have to wait for steady state?




Use a simplified model to estimate the time it takes the block to reach its steady state temperature.

**Assumptions:**

- Block has uniform temperature
- All heat is lost by convection

# Model of Start-up Transient (2)

A transient energy balance on the block gives

$$mc\frac{dT}{dt} = \dot{E}_{\mathrm{in}} - hA(T - T_a) \tag{5}$$

where $\dot{E}_{\mathrm{in}}$ is the electrical power input, $h$ is the heat transfer coefficient. Use a shifted temperature $\theta$, and the response time $\tau$

$$\theta = T - T_a \qquad \tau = \frac{mc}{hA} \tag{6}$$

to transform Equation (5) to dimensionless form

$$\frac{d\theta}{dt} = \frac{\dot{E}_{\mathrm{in}}}{mc} - \frac{\theta}{\tau} \tag{7}$$

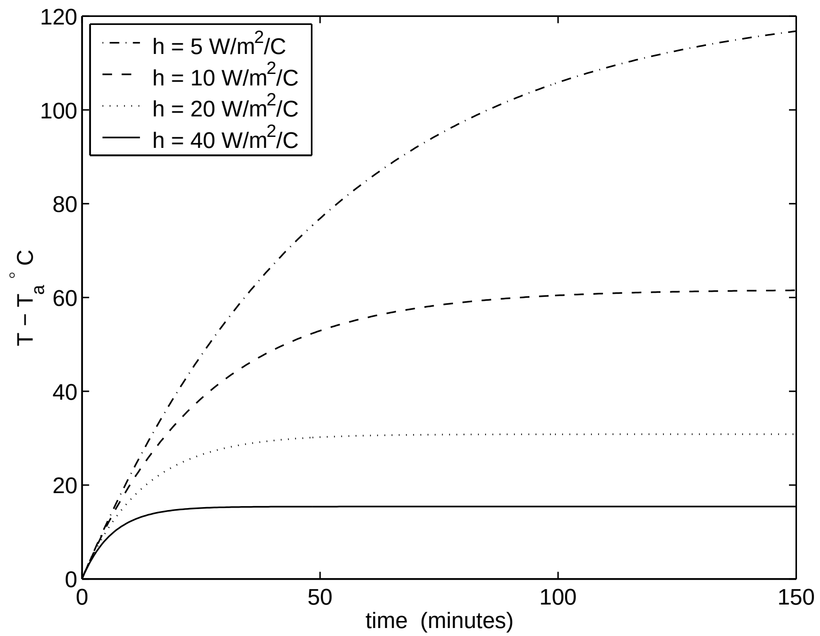
# Model of Start-up Transient (3)

With the initial condition

$$\theta(t = 0) = \theta_i = T_i - T_a$$

the solution to Equation (7) is

$$\theta = \frac{\dot{E}_{\rm in}}{hA} + \left(\theta_i - \frac{\dot{E}_{\rm in}}{hA}\right) e^{-t/\tau}$$



# Model of Start-up Transient (4)

Define $T_{\mathrm{ss}}$ as the steady state temperature

$$T_{\mathrm{ss}} = T_a + \frac{\dot{E}_{\mathrm{in}}}{hA}$$

and $t_p$ as the time for block to reach $p$ percent of its final temperature difference

$$\frac{T - T_a}{T_{\mathrm{ss}} - T_a} = \frac{p}{100}$$

With our model we compute

| $h$ $(\mathrm{W/m^2/^\circ C})$ | $\tau$ (min) | $T_{\mathrm{ss}}$ $(^\circ\mathrm{C})$ | $t_{50}$ (min) | $t_{90}$ (min) | $t_{99}$ (min) |
|---|---|---|---|---|---|
| 5 | 51 | 143 | 36 | 118 | 236 |
| 10 | 26 | 82 | 18 | 59 | 118 |
| 20 | 13 | 51 | 9 | 30 | 59 |
| 40 | 6 | 35 | 4 | 15 | 30 |

## Uncertainty Analysis (1)

*Suppose* that your data reduction equation had the form

$$h = f(V_h, V_{\rm sh}, R_{\rm sh}, A_s, T_a, T_w, T_{\rm bt}, T_{\rm bb}, k_b, \varepsilon) \tag{8}$$

$V_h =$ voltage across the heater
$V_{\rm sh} =$ voltage across the shunt resistor
$R_{\rm sh} =$ resistance of shunt resistor
$A_s =$ surface area of the block
$T_a =$ temperature of oncoming air
$T_w =$ temperature of duct walls
$T_{\rm bt} =$ temperature on the top surface of the board
$T_{\rm bb} =$ temperature on the bottom surface of the board
$k_b =$ thermal conductivity of the board
$\varepsilon =$ emissivity of the heated block

Data analysis involves creating a routine to compute $h$, and perturbation of all inputs to estimate $u_h$.

# Uncertainty Analysis (2)

Given the data reduction formula in Equation (8)

$$u_{h_{V_h}} = f(V_h + u_{V_h}, V_{\rm sh}, R_{\rm sh}, A_s, T_a, T_w, T_{\rm bt}, T_{\rm bb}, k_b, \varepsilon) - h$$

$$u_{h_{V_{\rm sh}}} = f(V_h, V_{\rm sh} + u_{V_{\rm sh}}, R_{\rm sh}, A_s, T_a, T_w, T_{\rm bt}, T_{\rm bb}, k_b, \varepsilon) - h$$

$$\vdots$$

$$u_h = \sqrt{(u_{h_{V_h}})^2 + (u_{h_{V_{\rm sh}}})^2 + \ldots}$$

where

| | |
|---|---|
| $u_{V_h}$ | the *total* uncertainty in the heater voltage |
| $u_{V_{\rm sh}}$ | the *total* uncertainty in the voltage across the shunt resistor |
| $\vdots$ | |

$u_{V_h}$, $u_{V_{\rm sh}}$, etc. depend on random error, instrument error, and calibration error.

# Shakedown

An end-to-end calibration is also called *first order replication*

- Used to verify equipment
- Make measurements of known quantities
  - ▷ Run experiment with power off: All temperature readings should be the same, i.e., room temperature
  - ▷ Perform benchmark experiments: Can you repeat experiments and obtain the same results published by others?
- One outcome is an estimate of random component of fixed error
  - ▷ Run the no-power experiment
  - ▷ Wait for system to come into steady state
  - ▷ Over time period of typical data collection, take 30 (or more) samples from each sensor
  - ▷ Compute average and standard deviation of sensor values
  - ▷ $u_{x_{\mathrm{random}}} = 2\sigma$ for 20:1 odds (95 % confidence)

## Relationship to Practical Problems

- Heaters as Prototype Devices
- Develop experience with multimode heat transfer
- Gain familiarity with complexity of measurements
- Gain experience building a thermal mock-up

## References

[1] R. J. Moffat. Uncertainty analysis. In K. Azar, editor, *Thermal Measurements in Electronic Cooling*, pages 45–80. CRC Press, Boca Raton, FL, 1997.