Pressure Transducer Calibration for M.E. Thermal Laboratory

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April 13, 2004

1 Transducer Identification

Table 1 provides the model numbers and serial numbers for the pressure transducers used in the Thermal Lab. The manufacturer, Omega Engineering, Inc., states that the transducers are accurate to within 0.25 percent of their full scale reading.

Table 1: Identification data for pressure transducers.

Range	Model Number	Serial Number
0–0.5 inch H_2O	PX653-0.5D5V	10523258
±2.5 inch H_2O	PX653-2.5BD5V	00204071
0–3 inch H_2O	PX653-03D5V	01212460
0–10 inch H_2O	PX653-10D5V	00100382

2 Simple Conversion Equation

Each of the pressure transducers has a linearized output from 1 to 5 Volts. The transducer produces 1 Volt when the pressure is at the minimum of the rated pressure range and 5 Volts when the pressure is at the maximum of the rated pressure range. Since the output is linear, the pressure at any intermediate voltage can be computed from

$$\frac{\delta p - \delta p_{\text{min}}}{\delta p_{\text{max}} - \delta p_{\text{min}}} = \frac{V_{\text{out}} - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}} \tag{1}$$

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where δp is the pressure *differential* across the ports. The output range of the transducers is fixed at $V_{\min} = 1\,\text{Volt}$ and $V_{\max} = 5\,\text{Volts}$. Thus, the measured pressure difference is

$$\delta p = \delta p_{\min} + \frac{1}{4}(\delta p_{\max} - \delta p_{\min})(V_{\text{out}} - 1). \tag{2}$$

Equation (2) can be used with very good accuracy to convert the pressure transducer output from voltage to pressure. The remainder of this document describes a slightly more accurate approach obtained from a curve fit of the calibration data supplied with the transducers.

3 Curve Fits

Calibration data for the pressure transducers supplied by Omega Engineering, Inc. is listed in Table 3. For each transducer a least squares line fit is obtained through the calibration data. The curve fit is of the form

$$\delta p = c_1 V + c_2 \tag{3}$$

where δp is the pressure applied to the transducer (all of the transducers measure pressure differential across two ports) and V is the voltage output of the transducer. Equation (1) can be algebraically manipulated into the form of Equation (3)[1]. A direct curve fit to the calibration data has the potential of being more accurate.

After a curve fit to Equation (3) is obtained, the residual of the fit is computed at each point in the calibration data set. The residual for data point i is

$$r_i = \delta p_{\text{fit},i} - \delta p_i$$

where $\delta p_{\text{fit},i} = c_1 V_i + c_2$. The residuals are plotted and the maximum residual

$$r_{\max} = \max |r_i|$$

is reported. The value of $r_{\max}$ is an indication of the largest error due to the curve fit of the calibration equation[2].

4 Total Uncertainty Estimates

The uncertainty in using the calibration equation is due to the accuracy of the sensor and the ability of the curve fit to represent the calibration data. The

[1]

$$c_1 = \frac{\delta p_{\max} - \delta p_{\min}}{V_{\max} - V_{\min}} \qquad c_2 = \frac{\delta p_{\min} V_{\max} - \delta p_{\max} V_{\min}}{V_{\max} - V_{\min}}$$

[2]The value of $r_{\max}$ depends on the amount of data in the calibration test. Increasing the number of calibration points would be almost certain to increase $r_{\max}$. The exception would be that the new data points would exactly lie along the curve fit for the smaller (original) data set. For a given number of calibration points $r_{\max}$ can be reduced by changing the fit function, for example, by fitting the data with a quadratic polynomial instead of a line.

manufacturer claims that the maximum transducer error is less than 0.25 percent of the full scale reading. A conservative interpretation of this error specification is that the calibration *data* supplied with the transducer is accurate to within

$$u_{\text{calibration}} = (\delta p_{\min} - \delta p_{\max}) \times \text{maximum full scale error specification.}$$

Let u_{fit} designate the error in using our conversion equation to predict the calibration data. If the curve fit from the preceding section is used, then $u_{\text{fit}} = r_{\max}$. The $u_{\text{calibration}}$ and u_{fit} errors are assumed to be uncorrelated, so that we can combine them using the root-sum-squared method. Thus, the total uncertainty in the pressure value obtained with the transducer is.

$$u_{\text{total}} = \sqrt{u_{\text{calibration}}^2 + u_{\text{fit}}^2} \tag{4}$$

If instead of the curve fit, we use the simple scaling in Equation (2), the uncertainty in the conversion equation will be different from u_{fit}. Let $r_{s,i}$ be error made when the simple scaling is used to predict the pressure for calibration point i

$$r_{s,i} = \delta p_i - \left[\delta p_{\min} + \frac{1}{4}(\delta p_{\max} - \delta p_{\min})(V_i - 1)\right] \tag{5}$$

The maximum error in using the simple scaling is

$$r_{s,\max} = \max |r_{s,i}|. \tag{6}$$

If the simple scaling is used, take $u_{\text{fit}} = r_{s,\max}$ in Equation (4).

The following section documents the details of curve fitting the calibration data for each transducer. Table 2 summarizes the errors for the curve fit and the calibration. In all cases the curve fit is the smallest error, i.e. $r_{\max} < r_{s,\max}$. The difference between $r_{\max}$ and $r_{s,\max}$ is small, however. The last column in Table 2 is the total uncertainty due to calibration and curve fit, i.e. $u_{\text{fit}} = r_{\max}$ in Equation (4). If $u_{\text{fit}} = r_{s,\max}$ is used instead of $u_{\text{fit}} = r_{\max}$, the total uncertainty would be only marginally larger.

Table 2: Curve fit and simple scaling errors for pressure transducers. Values of $r_{\max}$, $r_{s,\max}$, and $u_{\text{calibration}}$ have units of inches H_2O.

Serial No.	$r_{\max}$	$r_{s,\max}$	$u_{\text{calibration}}$	u_{total}
10523258	7.13×10^{-4}	7.50×10^{-4}	1.25×10^{-3}	1.49×10^{-3}
00204071	2.77×10^{-3}	3.75×10^{-3}	1.25×10^{-2}	1.28×10^{-2}
01212460	1.52×10^{-3}	3.00×10^{-3}	7.50×10^{-3}	7.65×10^{-3}
00100382	1.47×10^{-2}	2.25×10^{-2}	2.50×10^{-2}	2.90×10^{-2}

5 Calibration Details

In this section the least squares curve fits to the calibration data are presented. Each transducer is treated in a separate sub-section.

5.1 0–0.5 inch H_2O

Figure 1 is a plot of the calibration data and a least squares fit to the data for the 0–0.5 inch H_2O transducer. The equation of the fit is

$$\Delta p_{0.5} = 0.125039641376784V - 0.124734996011382 \qquad R^2 = 0.999996$$

Figure 2 is a plot of residual of the fit $\Delta p_{0.5} - \Delta p_{0.5,fit}$ for the points in the calibration data set. The maximum residual is less than 0.0007 inchH_2O, which is smaller than the uncertainty in the calibration data ($= 0.0025 \times 0.5 = 0.0013$). The combined uncertainty of the curve fit and the calibration is

$$u_{\text{tot}} = \sqrt{0.0013^2 + 0.0007^2} = 0.0015 \text{ inch}\,\mathrm{H_2O}$$

5.2 ± 2.5 inch H_2O

Figure 3 is a plot of the calibration data and a least squares fit to the data for the ± 2.5 inch H_2O transducer. The equation of the fit is

$$\Delta p_{25} = 1.25083247439622V - 3.75388723704910 \qquad R^2 = 0.999999$$

Figure 4 is a plot of residual of the fit $\Delta p_{25} - \Delta p_{25,fit}$ for the points in the calibration data set. The maximum residual is less than 0.0028 inchH_2O, which is less than the uncertainty in the calibration data ($= 0.0025 \times 2.5 = 0.0063$). The combined uncertainty of the curve fit and the calibration is

$$u_{\text{tot}} = \sqrt{0.0028^2 + 0.0063^2} = 0.0069 \text{ inch}\,\mathrm{H_2O}$$

5.3 0–3 inch H_2O

Figure 5 is a plot of the calibration data and a least squares fit to the data for the 0–3 inch H_2O transducer. The equation of the fit is

$$\Delta p_{30} = 0.750664358028092V - 0.753430174833871 \qquad R^2 = 0.999999$$

Figure 6 is a plot of residual of the fit $\Delta p_{30} - \Delta p_{30,fit}$ for the points in the calibration data set. The maximum residual is less than 0.0016 inchH_2O, which is less than the uncertainty in the calibration data ($= 0.0025 \times 30 = 0.0075$). The combined uncertainty of the curve fit and the calibration is

$$u_{\text{tot}} = \sqrt{0.0016^2 + 0.0075^2} = 0.0077 \text{ inch}\,\mathrm{H_2O}$$

5.4 0–10 inch H_2O

Figure 7 is a plot of the calibration data and a least squares fit to the data for the 0–10 inch H_2O transducer. The equation of the fit is

$$\Delta p_{10} = 2.50600948477838V - 2.51613612227667 \qquad R^2 = 0.999991$$

Figure 8 is a plot of residual of the fit $\Delta p_{10} - \Delta p_{10,fit}$ for the points in the calibration data set. The maximum residual is less than 0.015 inchH_2O, which of the same order as the error in the calibration data ($= 0.0025 \times 10 = 0.025$). The combined uncertainty of the curve fit and the calibration is

$$u_{\text{tot}} = \sqrt{0.0015^2 + 0.0025^2} = 0.029 \text{ inch}\, H_2O$$

5.5 Calibration Computations

The curve fits to the calibration data are performed with the `calibratePtrans` in Listing 1. By default, `calibratePtrans` fits a straight line to the calibration data for each transducer. Higher order polynomials can be used by supplying an optional input argument to `calibratePtrans`.

Running `calibratePtrans` for least squares line fit produces the plots in Figure 1 through Figure 8 on the following pages. The text output from `calibratePtrans` is

```
>> calibratePtrans

Calibration data for model PX653-0.5D5V,  S/N 10523258
    c = 1.25039641376785e-001    -1.24734996011383e-001
    R2 =  0.999995601
    Maximum residual of fit is   7.133e-004 inch H2O
    Maximum error in the simplistic fit is   7.500e-004 (inch H20)
    Manufacturers stated accuracy is   1.250e-003 (inch H20)
    Total uncertainty is   1.439e-003 (inch H20)

Calibration data for model PX653-2.5BD5V,  S/N 00204071
    c = 1.25083247439623e+000    -3.75388723704912e+000
    R2 =  0.999998869
    Maximum residual of fit is   2.777e-003 inch H2O
    Maximum error in the simplistic fit is   3.750e-003 (inch H20)
    Manufacturers stated accuracy is   1.250e-002 (inch H20)
    Total uncertainty is   1.280e-002 (inch H20)

Calibration data for model PX653-03D5V,  S/N 01212460
    c = 7.50664358028094e-001    -7.53430174833876e-001
    R2 =  0.999999312
    Maximum residual of fit is   1.523e-003 inch H2O
    Maximum error in the simplistic fit is   3.000e-003 (inch H20)
    Manufacturers stated accuracy is   7.500e-003 (inch H20)
    Total uncertainty is   7.653e-003 (inch H20)

Calibration data for model PX653-10D5V,  S/N 00100382
    c = 2.50600948477839e+000    -2.51613612227668e+000
    R2 =  0.999990865
    Maximum residual of fit is   1.465e-002 inch H2O
    Maximum error in the simplistic fit is   2.250e-002 (inch H20)
    Manufacturers stated accuracy is   2.500e-002 (inch H20)
    Total uncertainty is   2.898e-002 (inch H20)
```

Repeating the curve fits with a quadratic polynomial gives

```
>> calibratePtrans(2)

Calibration data for model PX653-0.5D5V,  S/N 10523258
    c = -2.10751467520694e-004    1.26244189609561e-001    -1.26090662310183e-001
    R2 =  0.999999683
    Maximum residual of fit is   1.785e-004 inch H2O
    Maximum error in the simplistic fit is   7.500e-004 (inch H20)
    Manufacturers stated accuracy is   1.250e-003 (inch H20)
    Total uncertainty is   1.263e-003 (inch H20)

Calibration data for model PX653-2.5BD5V,  S/N 00204071
    c = -4.17498129397241e-004    1.25333908788998e+000    -3.75709367282575e+000
    R2 =  0.999999055
    Maximum residual of fit is   1.670e-003 inch H2O
    Maximum error in the simplistic fit is   3.750e-003 (inch H20)
    Manufacturers stated accuracy is   1.250e-002 (inch H20)
    Total uncertainty is   1.261e-002 (inch H20)

Calibration data for model PX653-03D5V,  S/N 01212460
    c = 1.24743659525342e-004    7.49951117193317e-001    -7.52626661109283e-001
    R2 =  0.999999351
    Maximum residual of fit is   1.577e-003 inch H2O
    Maximum error in the simplistic fit is   3.000e-003 (inch H20)
    Manufacturers stated accuracy is   7.500e-003 (inch H20)
    Total uncertainty is   7.664e-003 (inch H20)

Calibration data for model PX653-10D5V,  S/N 00100382
    c = -2.30773371377629e-004    2.50732808644500e+000    -2.51762128459506e+000
    R2 =  0.999990877
    Maximum residual of fit is   1.455e-002 inch H2O
    Maximum error in the simplistic fit is   2.250e-002 (inch H20)
    Manufacturers stated accuracy is   2.500e-002 (inch H20)
    Total uncertainty is   2.893e-002 (inch H20)
```

The quadratic provides a significant improvement in the curve fit only for the 0.05 inchH_2O transducer. However, the data in Table 2 shows that in all cases the error in the manufacturer's calibration is greater than the error in the curve fit to the calibration data. Thus, using the quadratic polynomial in the curve fit is of no advantage.

Table 3: Calibration data for Omega pressure transducers used in the flow bench.

± 2.5 inch H_2O pressure transducer, Omega Model PX653-2.5D5V

V_{out} (volts)	Δp inch H_2O
3.001	0.000
3.999	1.250
5.002	2.500
3.999	1.250
3.001	0.000
2.003	−1.250
1.002	−2.500
2.003	−1.250
3.000	0.000

0–10 inch H_2O pressure transducer, Omega Model PX653-10D5V

V_{out} (volts)	Δp inch H_2O
1.002	0.000
2.006	2.500
3.001	5.000
3.991	7.500
5.000	10.000
3.991	7.500
3.000	5.000
2.005	2.500
1.002	0.000

0–0.5 inch H_2O pressure transducer, Omega Model PX653-0.5D5V

V_{out} (volts)	Δp inch H_2O
1.000	0.000
1.997	0.125
2.994	0.250
3.995	0.375
5.002	0.500
3.995	0.375
2.994	0.250
1.996	0.125
1.000	0.000

0–3 inch H_2O pressure transducer, Omega Model PX653-03D5V

V_{out} (volts)	Δp inch H_2O
1.003	0.000
2.004	0.750
3.003	1.500
4.000	2.250
5.001	3.000
3.999	2.250
3.003	1.500
2.003	0.750
1.003	0.000

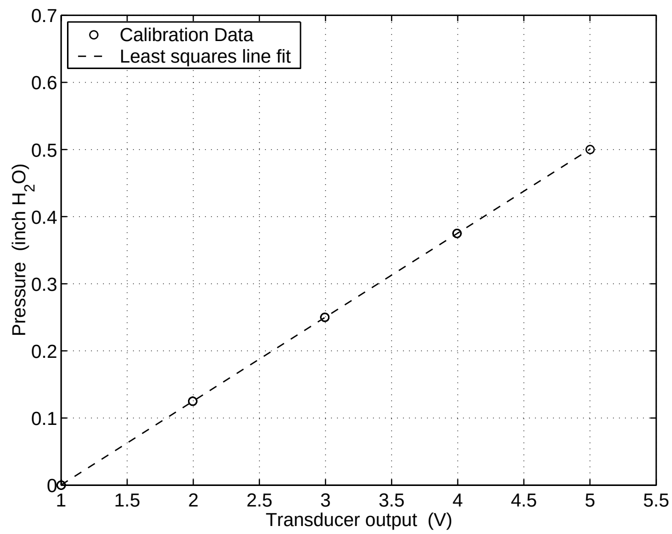

Figure 1: Calibration of the 0–0.5 inch H_2O pressure transducer, Omega Model PX653-0.5D5V.

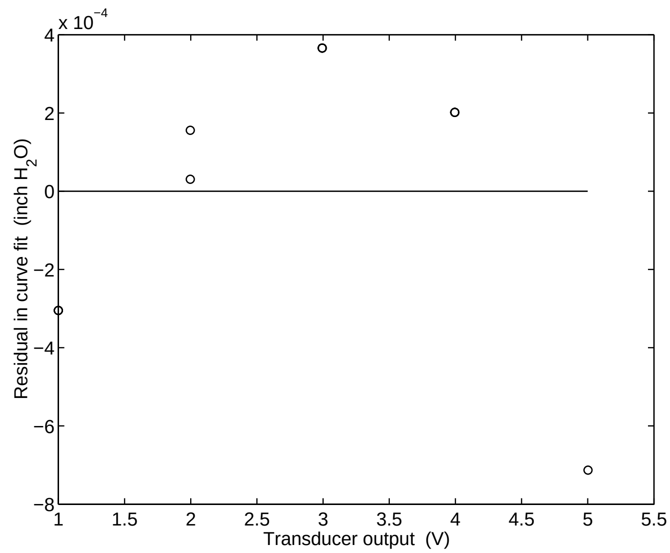

Figure 2: Curve fit residuals for the calibration data of the 0–0.5 inch H_2O pressure transducer, Omega Model PX653-0.5D5V.

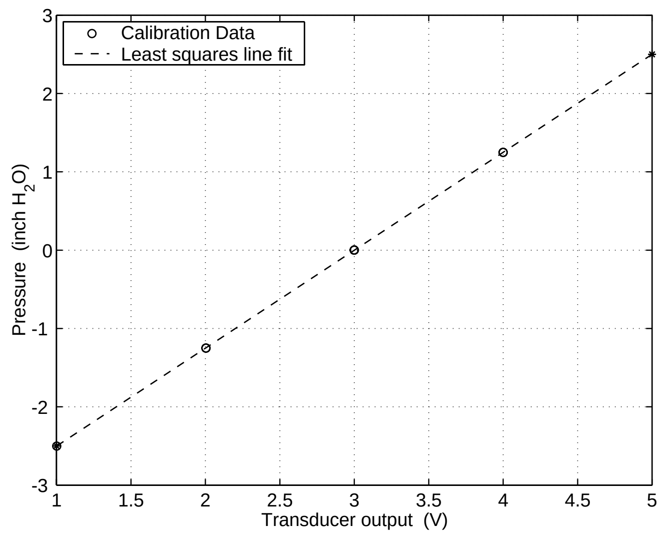

Figure 3: Calibration of the ±2.5 inch H_2O pressure transducer, Omega Model PX653-2.5D5V.

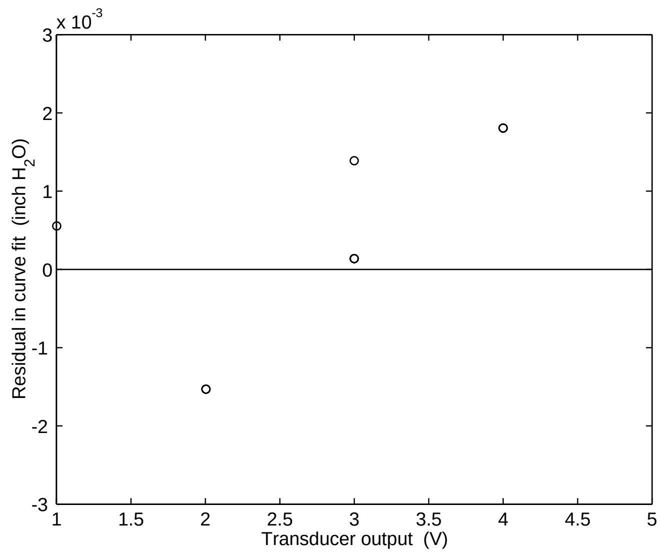

Figure 4: Curve fit residuals for the calibration data of the ±2.5 inch H_2O pressure transducer, Omega Model PX653-2.5D5V.

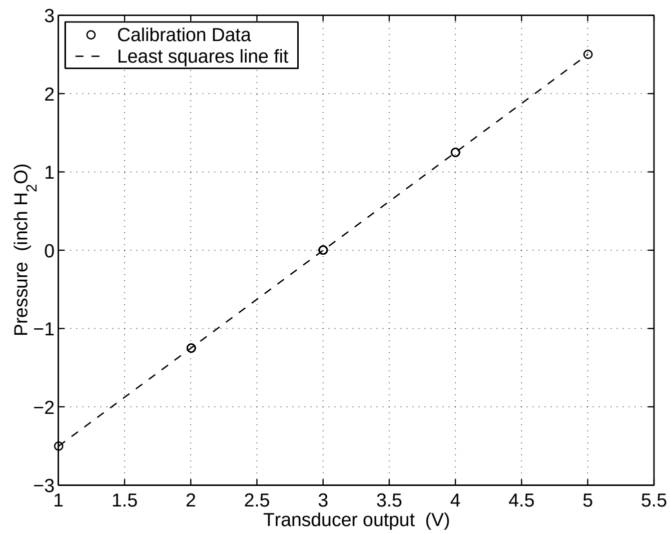

Figure 5: Calibration of the 0–3 inch H_2O pressure transducer, Omega Model PX653-03D5V.

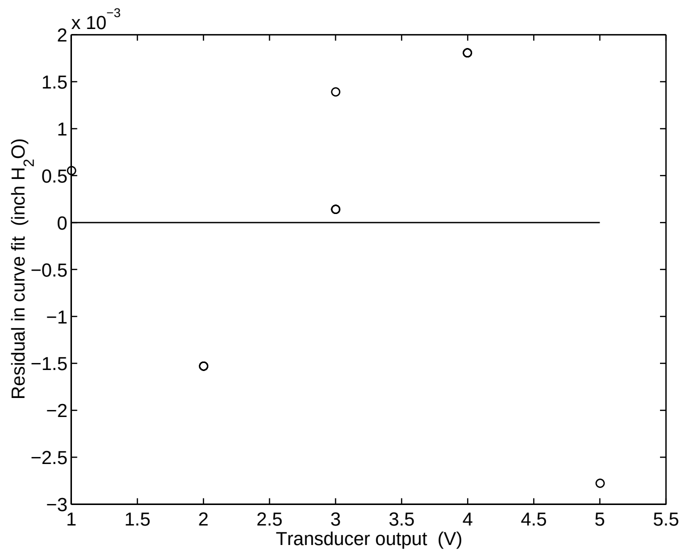

Figure 6: Curve fit residuals for the calibration data of the 0–3 inch H_2O pressure transducer, Omega Model PX653-03D5V.

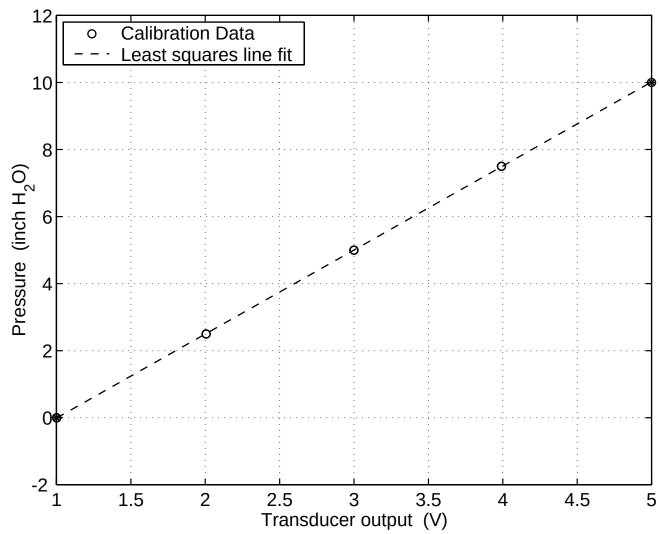

Figure 7: Calibration of the 0–10 inch H_2O pressure transducer, Omega PX653-10D5V.

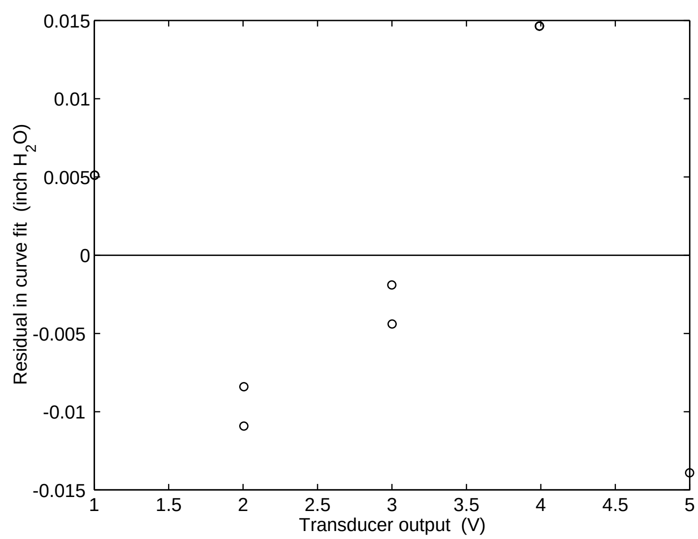

Figure 8: Curve fit residuals for the calibration data of the 0–10 inch H_2O pressure transducer, Omega PX653-10D5V.

```
function calibratePtrans(n)
% calibratePtrans  Least squares line fite of calibration data for pressure transducer

if nargin<1, n=1;  end
% --- Data from Calibration Report:  Omega DP transducer:  0 <= dp <= 0.5 inch H2O
dpw  = [ 0    0.125  0.250  0.375  0.500  0.375  0.250  0.125   0];  % Delta p, inch H2O
vout = [ 1.0  1.9970 2.994  3.995  5.002  3.995  2.994  1.996  1.0]; % Volts
makeFit(vout,dpw,n,'model PX653-0.5D5V,  S/N 10523258',0,0.5);

% --- Data from Calibration Report:  Omega DP transducer |dp| <= 2.5 inch H2O
dpw  = [0 1.25  2.5  1.25  0  -1.25  -2.5  -1.25  0];   %  Delta p,  inch H2O
vout = [3.001  3.999  5.002  3.999  3.001  2.003  1.002  2.003  3.000];  % volts
makeFit(vout,dpw,n,'model PX653-2.5BD5V,  S/N 00204071',-2.5,2.5);

% --- Data from Calibration Report:  Omega DP transducer:  0 <= dp <= 3 inch H2O
dpw =  [ 0     0.750  1.500  2.250   3.000  2.250  1.500  0.750  0 ];    % Delta p, inch H2O
vout = [1.003  2.004  3.003  4.000   5.001  3.999  3.003  2.003  1.003]; % Volts
makeFit(vout,dpw,n,'model PX653-03D5V,  S/N 01212460',0,3);

% --- Data from Calibration Report:  Omega DP transducer:  0 <= dp <= 10 inch H2O
dpw  = [0.00  2.5  5.0  7.5  10  7.5  5.0  2.5  0.0];   %  Delta p,  inch H2O
vout = [1.002  2.006  3.001  3.991  5.000  3.991  3.000  2.005  1.002];  % volts
makeFit(vout,dpw,n,'model PX653-10D5V,  S/N 00100382',0,10);
```

Listing 1: Main Matlab program to supply calibration data to the `makeFit` routine for the pressure transducers used in the lab.

```
function makeFit(v,dp,n,name,pmin,pmax)
% makeFit  Perform the pressure transducer curve fit, plot and print the results
%
% Synopsis:  makeFit(v,dp,n,name,pmin,pmax)
%
% Input: v  = voltage output of transducer
%        dp = pressure differential applied to the transducer
%        n  = (optional) degree of the polynomial curve fit.  Default: n=1;
%        name = (optional) string used to identify the data set in print out
%                and plots
%        pmin, pmax = (optional) minimum and maximum range of pressures for
%                     the transducer.  These values are used to compute the
%                     error limits due to using the simplistic scaling of
%                     the output, and the manufacturers limit of calibration
%
% Output: print out of curve fit coefficients

if nargin<3,  n=1;  end
if nargin<4,  name='no name';  end

c = polyfit(v,dp,n);     %  linefit is from NMM Toolbox
vfit = [min(v) max(v)];  %  evaluate fit at these voltages
dpfit = polyval(c,vfit); %  c(1)*vfit + c(2);
hc = figure;  set(hc,'Name',name);   % new window with 'name' in title bar
plot(v,dp,'o',vfit,dpfit,'--');
grid on;
xlabel('Transducer output  (V)');  ylabel('Pressure  (inch H_2O)');
legend('Calibration Data','Least squares line fit',2);
fprintf('\nCalibration data for %s\n\tc = ',name);
fprintf('%18.14e    ',c);  fprintf('\n');

% --- Compute and plot residuals at data points in the calibration set
r = dp - polyval(c,v);  % ( c(1)*v + c(2) );
R2 = 1 - (norm(r)/norm(dp-mean(dp)))^2;
fprintf('\tR2 = %12.9f\n',R2);
rmax = max(abs(r));
fprintf('\tMaximum residual of fit is %12.3e inch H2O\n',rmax);
hr = figure;  set(hr,'Name',name);   %  new window with 'name' in title bar
plot(v,r,'o',[1 5],[0 0],'k-')
xlabel('Transducer output  (V)');  ylabel('Residual in curve fit  (inch H_2O)');

% --- Compare the curve fit to a simplistic linear scaling of the transducer range
%     dps is the pressure difference predicted by
%             (dps-pmin)/(pmax-pmin) = (vout-vmin)/(vmax-vmin)
if nargin>4
  vmax = 5;  vmin = 1;
  dps = pmin + (pmax-pmin)*(v-vmin)/(vmax-vmin);
  rs = dp - dps;
  fprintf('\tMaximum error in the simplistic fit is %12.3e (inch H2O)\n',max(abs(rs)));
  eman = 0.0025*(pmax-pmin);   %  absolute error based on full scale error specification
  fprintf('\tManufacturers stated accuracy is %12.3e (inch H2O)\n',eman);
  utot = sqrt(eman^2 + rmax^2);
  fprintf('\tTotal uncertainty is %12.3e (inch H2O)\n',utot);
end
```

Listing 2: MATLAB subprogram used to create calibration curve fits and plots.