

**Quick Questions**

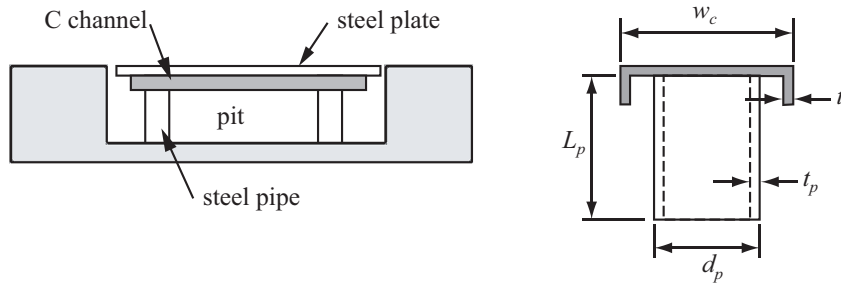
In the textbook problem 5.21, the following formula is given for the optimum damping ratio  $\xi$  of a spring-mass-damper system

$$\cos \left[ 4\xi \sqrt{1 - \xi^2} \right] = -1 + 8\xi^2 - 8\xi^4 \quad (\star)$$

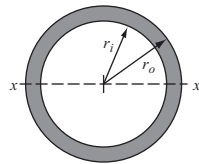
1. **(5 points)** Rearrange Equation  $(\star)$  in at least two different ways to obtain two functions of the form  $f_1(\xi) = 0$  and  $f_2(\xi) = 0$ .
2. **(5 points)** Write an m-file for each of the  $f_1(\xi)$  and  $f_2(\xi)$  functions you derived in the preceding problem. One function should evaluate  $f_1(\xi)$  and the other function should evaluate  $f_2(\xi)$ .
3. **(10 points)** Write a MATLAB function that plots Equation  $(\star)$  (or some useful form of it) in a way that allows you to estimate the location of the root. Give numerical values for a bracket on the root.
4. **(10 points)** Use the `bisect` function from the NMM Toolbox to find the root of Equation  $(\star)$ . *Do not* modify any code in `bisect`.

## Comprehensive Questions

As part of your summer internship, you are given the job to design a temporary bridge over a pit on a construction site. The bridge is to be constructed from steel plate supported by four legs made from large pipe as shown in the left side of the following schematic. The load from the steel plate is distributed to the legs by C channel. The right side of the schematic shows the relevant dimensions of the C channel and pipe.



The following diagram gives some useful formulas for geometric properties of a circular tube



$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$A = \pi(r_o^2 - r_i^2)$$

$$k = \frac{1}{2}\sqrt{r_o^2 + r_i^2}$$

5. (a) **(10 points)** Write an m-file function called `pipeLoadDelta` with the following function definition. The body of the `pipeLoadDelta` function should call the `beamPcr` function from the solution to Problem Set 2. *Do not* modify the code in `beamPcr`.

```
function dP = pipeLoadDelta(t,d,P,Leff,E,Sy)
% pipeLoadDelta Difference between applied and critical loads for a pipe in compression
%
% Synopsis: dP = pipeLoadDelta(t,d,P,Leff,E,Sy)
%
% Input: t = pipe wall thickness
%        d = pipe diameter
%        P = applied load
%        Leff = effective length of the pipe
%        E = elastic modulus of the pipe material
%        Sy = yield stress of pipe material
%
% Output: dP = Pcr - P = amount by which the critical buckling load exceeds
%          the applied load for the pipe.
%
% Note: The user is responsible for insuring that all input variables
%       have consistent units
```

- (b) **(20 points)** Write another m-file that calls `pipeLoadDelta` for a range of  $t$  values and  $d$  values. Plot  $\Delta P$  versus  $t$  for a set of  $d$  values. Use the properties of structural steel ( $E = 200$  GPa,  $S_y = 250$  MPa),  $L = 1$  m,  $0.5 \leq t \leq 2$  cm and an applied load of 1.25 MN. Make sure that  $d$  does not exceed  $w_c - 3t_c$ . Assume that the physical constraints are best modeled by clamped-free boundary conditions, i.e.,  $L_{\text{eff}} = 2.1L$ . Use  $w_c = 30$  cm,  $t_c = 1.3$  cm.
- (c) **(5 points)** Use the plot from part (b) to suggest at least two combinations of  $t$  and  $d$  that would avoid buckling under a load of 1.25 MN.