Bare Essentials

At the end of this chapter you should be able to

1. Transform equations written in the natural variables of an applied problem to the canonical $Ax = b$ of linear algebra.
2. Explain the condition of consistency in terms of linear combinations of column vectors.
3. Explain the condition of singularity of an $n \times n$ matrix in terms of linear independence.
4. Express matrix rank as a measure of linear independence.
5. Relate rank of the coefficient matrix to the consistency of a $n \times n$ system of equations.
6. Write the formal solution to $Ax = b$.
7. Explain why it is not a good idea to use the formal solution as a computational procedure for solving $Ax = b$.
8. Describe the most efficient procedures for solving $Lx = b$ or $Ux = b$ when L is lower triangular and U is upper triangular.
9. Name the solution algorithm most commonly used for solving $Ax = b$.
10. Write the equation that defines the residual vector.
11. Describe the significance of $\kappa(A)$ on the reliability of the numerical solution to $Ax = b$.
12. Describe the significance of $\|r\|$ for a well-conditioned A.
13. Describe the significance of $\|r\|$ for a ill-conditioned A.
14. Describe the reason for pivoting. Is pivoting a remedy for ill-conditioned systems?

To perform basic solutions of linear systems with Matlab you will need to

15. Assign the elements of matrix `A`, and vector `b`, for a system of equations.
16. Write a compact (one line) statement that uses the recommended method for solving $Ax = b$, given that A and b are already assigned to Matlab variables.
17. Compute $\|r\|$ of a system given that A, x, and b are already assigned to Matlab variables.

An Expanded Core of Knowledge

After mastering the bare essentials you should move on to a deeper understanding of the fundamentals. Doing so involves being able to

1. Describe the qualitative relationship between the magnitude of $\kappa(A)$ and the singularity of A.
2. Estimate the number of correct significant digits in the numerical solution to $Ax = b$ given values of ε_m and $\kappa(A)$.
3. State conditions required for a successful LU factorization of A.
4. Write (describe) a procedure for solving $Ax = b$ given an LU factorization of A.
5. State conditions required for a successful Cholesky factorization of A.
6. Write (describe) a procedure for solving $Ax = b$ given a Cholesky factorization of A.

To perform more advanced solutions of linear systems with MATLAB you will need to

7. Write the preferred expression for solving $Lx = b$ or $Ux = b$ when L is lower triangular and U is upper triangular. What algorithm does MATLAB select to implement the solution for these systems?
8. Use MATLAB and the LU factorization of A to solve several systems of equations that have the same A and a sequence of different b.
9. Use MATLAB and a Cholesky factorization of A to solve several systems of equations that have the same A and a sequence of different b.
10. Implement solutions of nonlinear systems of equations with successive substitution.
11. Implement solutions of nonlinear systems of equations with Newton's method.

Developing Mastery

Working toward mastery of solving systems of equations you will need to

1. Given a variety of $m \times n$ system of equations, where m is not necessarily equal to n, describe the method used by the `\` operator to solve $Ax = b$.
2. Given L, U, and permutation matrix P from an LU factorization of A, apply these to solve $Ax = b$. Specifically, use the P appropriately.
3. Explain how MATLAB uses the L and U factors returned from the `lu` command to solve $Ax = b$ *without* explicitly requiring P.
4. List the order of magnitude work estimates for Gaussian elimination with back substitution, LU factorization, and Cholesky factorization.